Flexible Application of Infinitesimal Analysis in Mathematical Learning

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ABSTRACT

The method of infinitesimal analysis is one of the important means to solve calculus problems. It is particularly crucial to enable students to flexibly use the method of infinitesimal analysis to solve practical problems during the learning process. This article reveals the application of infinitesimal analysis through practical problems such as calculating the area of planar shapes, the arc length of curves, and the volume of solids, based on the experience accumulated in daily teaching, which enables students to have a deeper understanding of infinitesimal analysis and apply what they have learned. Using infinitesimal analysis can solve some complex problems in familiar ways and simplify the problems.

Keywords: *Infinitesimal analysis, Curve, Curved surface.*

1. INTRODUCTION

Infinitesimal analysis is an important tool that combines mathematical theory with practice. Through a deep understanding and analysis of the method, people can make complex problems easier to handle when dealing with practical problems. With the development of mathematics, the position of infinitesimal analysis in practical applications is becoming increasingly important. The in-depth study of calculus not only requires students to master some specific calculation formulas, but also to learn how to analyze and solve problems through thinking and methods. By cultivating students' thinking and application abilities during the teaching process, they can better play a greater role in solving practical problems

At the beginning of the chapter on the application of definite integrals, the teachers can lead the students to recall the solution to the area of a trapezoidal curve. Through four steps: segmentation, approximation, summation, and limit calculation, the area of the curve shape can be obtained. However, any practical problem cannot be solved through cumbersome steps every time, so simplifying the steps is imperative, and the infinitesimal analysis has emerged. Infinitesimal analysis is a common method of solving problems in mathematics and physics. Starting from the perspective of performing infinitesimal analysis on a certain thing, it achieves a solution to the complete limitations of that thing. It is to split a large amount of data, find a simple rule, and then accumulate and continuously sum up to achieve a whole. Allowing students to flexibly apply such methods will help enhance their ability to re explore known patterns, thereby generating positive effects such as consolidating learned knowledge, enhancing understanding, and enhancing abilities.

2. METHOD ANALYSIS OF INFINITESIMAL ANALYSIS

In the previous study of definite integrals, students have become proficient in achieving their goals through four steps. However, simplifying the steps and directly using definite integrals to represent the results still presents some confusion. During the teaching process, teachers should guide students on how to infer the process from the final conclusion. It is obvious that the representation of definite integrals is very close to the second step. Teachers need to tell students that the second step of the process is actually a partial compression of the whole, and its representation is very similar to the final conclusion. Any small curved trapezoid can be approximated as a small rectangle, and its area is not related to the selection of any point between cells. The left or right endpoint between cells can be taken. Therefore, the steps of the infinitesimal analysis can be summarized by analyzing the area of the trapezoid with curve side A enclosed by the continuous curve $f(x)$ and the straight line $x=a, x=b, x$ axis.

The first step is to find the element: project the plane shape onto the coordinate axis to determine the integral variable. If it is projected onto the x axis, the projection interval is the $[a,b]$. Selecting any small area within a large interval, the approximate value of the corresponding graphic area for this small area is what is needed. In approximate flat graphics, it is easier to calculate the area of a small rectangle than to calculate the area of other graphics. Its length is dx, its height is $f(x)$, and the approximate area can be expressed as $f(x)dx$, which it is called the area element [1], denoted as $dA = f(x)dx$.

Step 2: quadrature: the area element on the $[a,b]$ of the entire variation interval is integrated to obtain the expression for the required area [2]: $A=\int_a^b dA=\int_a^b f(x)dx$.

Through the above analysis, it can be clearly found that the key to the infinitesimal analysis is to first determine the interval and then obtain the element.

3. USING INFINITESIMAL ANALYSIS TO CALCULATE THE AREA OF A PLANE SHAPE

If the planar region is enclosed by two continuous curves $y=f(x)$ (the curve above) and $y=g(x)$ (the curve below) on the interval $[a,b]$, the question is to analyze how to use the infinitesimal analysis to determine its area. During the teaching process, students are asked the question of whether the desired graph is projected onto the x axis or onto the y axis, and where to determine the integral variable. From the function curve, it can be seen that taking the integral variable on the x axis of the mathematical formul is more appropriate. After determining the interval of x, according to the previous analysis, any small interval within the determined interval $[a,b]$ is taken. The left endpoint coordinate of the small interval is x, and the right endpoint coordinate is $x+dx$. The approximate area corresponding to this interval is the rectangular area. The length of a rectangle is dx, and the height is the distance between two curves, which is the difference between the vertical coordinates of the upper curve corresponding to the left endpoint and the vertical coordinates of the lower curve. This gives the $dA=[f(x)-g(x)]dx$ for the area element.

The area obtained by integrating $[a,b]$ on both sides simultaneously is: $A=\int_a^b f(x)-g(x)dx$. An example can be used to verify the application of this formula, and then calculate the area of the graph enclosed by two parabolic $y^2=x$ and $y=\frac{1}{8}x^2$. After analysis, it can be clearly seen that the enclosed figure can be projected onto both the mathematical formula axis and the mathematical formula axis. Assuming to take x for the integral variable, it will found that the projection interval is $[0,4]$, and the corresponding area between any small cells in $[0,4]$ can be calculated, thus obtaining $dA=(\sqrt{x}-\frac{1}{8}x^2)dx$ for area element. Integrating on an interval yields $A=\int_0^4 (\sqrt{x}-\frac{1}{8}x^2)dx$ for the desired area, which is consistent with the formula obtained using infinitesimal analysis.

If a planar figure is represented by a polar coordinate equation, how can people calculate the area of a trapezoid with curved edges in a polar coordinate system? When teaching this section, most students are not familiar with polar coordinates, so it is difficult to shift their thinking from Cartesian coordinates to polar coordinates. Therefore, when explaining, teachers can tell students to transfer the work done in Cartesian coordinates to polar coordinates. As long as it meets the requirements of polar coordinates, such as $x=a$ of Cartesian coordinates, it corresponds to $\theta=\alpha$ of polar coordinates. But in polar coordinates, $\theta=\alpha$ is a ray from the pole and is not a straight line perpendicular to the coordinate axis. Following this approach, students can calculate $r=f(\theta),(\alpha\le\theta\le\beta)$ for a plane curve, and $f(\theta)$ is continuous in $[\alpha,\beta]$, and the surface product enclosed by two rays $\theta = \alpha$ and $\theta = \beta$. According to the method of differential analysis, the integration interval is first determined. Obviously, the boundary of the graph is two rays starting from the poles, which determines the integration interval as $[\alpha, \beta]$. $[\theta, \theta + d\theta]$ for any given small interval can be approximated as a small isosceles triangle or a small sector, forming a small shape. It is much simpler to calculate the area of a sector than the area of a triangle based on known conditions. Therefore, the corresponding curved sector area between the communities is approximately the small sector area with $d\theta$ as the angle and $r = f(\theta)$ as the radius, and the area element is obtained as $dA = \frac{1}{2}[f(\theta)]^2 d\theta$. By integrating on interval $[\alpha, \beta]$, the area of the curved sector obtained is [2]: $A = \int_\alpha^\beta \frac{1}{2}[f(\theta)]^2 d\theta$.

4. USING INFINITESIMAL ANALYSIS TO CALCULATE ARC LENGTH

There is a known formula for calculating the length of a line segment in middle school mathematics, but the distance formula between two points only applies to straight lines and not curves. If a curve $y = f(x)$ mathematical formula given now has a continuous derivative on an interval $[a,b]$, asking students to calculate the arc length of the curve, many of them are confused and do not know how to start. When teaching this part of the knowledge, teachers ask students to recall the geometric meaning of differentiation. dy is the amount of change of the curve dy in the ordinate of a tangent line at a point, and Δy is the amount of change in the ordinate of a curve at that point. The geometric application of differentiation is to approximate the change in the y-axis of a tangent line as a substitute for the change in $f(x)$ of a function. Switching this approach to horizontal thinking means that when Δx is sufficient, this curve arc can be approximated as the corresponding straight line length. In fact, this is also the approach of constructing microelements using the method of infinitesimal analysis to calculate the curve arc length. Therefore, during the teaching period, guiding students to open up their minds and connect them with the previous knowledge points not only consolidates old knowledge but also expands new ideas. Therefore, based on existing conditions and combined with the method of differential analysis, any small cell $[x, x + dx]$ can be selected within the $[a,b]$, and the length of the corresponding small tangent line segment can be used to replace the length of the curve arc between small cells. The the tangent line segment is placed into a right angled triangle, and the two right angled tables represent dx and dy, respectively. The included angle is determined by the slope of the tangent line, and $\sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + y'^2}dx$ for the slope of a right angled triangle is the length of the small tangent line segment, which is the arc length element. By performing a definite integral on $[a,b]$ of the variation interval, the arc length of the curve being calculated is obtained: $L = \int_a^b \sqrt{1 + y'^2}dx$ 。

5. USING INFINITESIMAL ANALYSIS TO CALCULATE VOLUME

5.1 *Finding the Volume of a Solid with a Known Cross-sectional Area*

During the teaching process, it was emphasized to the students that the most important aspect of infinitesimal analysis is to determine the integration variables, determine the integration interval, and facilitate the final integration. Therefore, when calculating the volume of a solid with a known cross-sectional area, students are guided to apply this approach. Assuming that the cross-sectional area is a function of x, and that the area function $A(x)$ is a continuous function on $[a,b]$, the question is to find $[a,b]$ for the volume of the solid.

Seeing this three-dimensional object, during the teaching process, teachers ask questions to guide students to think. Does the volume change when the object is placed in different positions in the space? The classmates unanimously answered that they would not. So since the placement does not affect the volume value, should this solid be placed in an appropriate positio to calculate? The primary problem solved by the differential analysis method is to determine the integral variables, and how to place them so that the projection can be easily obtained. It is obvious that placing a solid perpendicular to x axis or perpendicular to y axis makes it easier to find the projection coordinates. Traditionally, the solid perpendicular is placed to x axis, so the integration interval is $[a,b]$. The next is to take any interval $[x, x+dx]$ from the interval $[a,b]$. At this point, teachers and students observe what the corresponding solid shape is between this small area, because the faces surrounding the solid are curved surfaces. Therefore, the shape of this small part of the solid is also irregular. The construction of the infinitesimal analysis is actually to find an approximate value. So, returning to the small solid, which type of solid is simpler to find its volume? When Δx is very small, approximating this small piece of solid as a cylinder for volume is the simplest. The bottom area of the solid is $A(x)$, and the height is Δx. Therefore, the corresponding volume of the small cylinder is approximated as $dV = A(x)dx$. Then, students can integrate on interval $[a,b]$ to obtain the integral expression of the volume of the solid: $V = \int_a^b A(x)dx$.

Through the above analysis, students have become more familiar with the steps of infinitesimal analysis and have derived the volume formula. This makes it more convenient to solve practical problems. For example, a common example in daily life is to find the volume of a solid enclosed by two cylinders $x^2 + y^2 = a^2, x^2 + z^2 = a^2$. From the equation, people can know that both cylinders are special. If they are placed in a spatial Cartesian coordinate system, it is obvious that all eight hexagram limits have symmetry and equality. Therefore, the first hexagram limit is the only to be considered. Firstly, this part is projected onto x axis in a three-dimensional manner, with the variation interval being $[0,a]$. Taking any small interval in $[0,a]$, if the cross-section of the solid cut by a plane perpendicular to the axis is a square with a side length equal to $\sqrt{a^2 - x^2}$, then $A(x) = a^2 - x^2$ for the cross-sectional area of the solid cut by the plane. By integrating on the projection interval, the integral expression for the volume of the solid can be obtained.

5.2 *Finding the Volume of a Rotating Body*

During the teaching process, consideration is given to the volume of a solid rotating around a coordinate axis. If rotating around x axis, then the entire solid is projected onto x axis. Taking any small cell $[x, x+dx]$ in $[a,b]$, the cross-sectional area is the area of a circle with a radius equal to $f(x)$, and the corresponding volume element is $dV = \pi f^2(x)dx$. Then, students can integrate on interval $[a,b]$ to obtain the integral expression of the volume of the solid: $V = \pi\int_a^b f^2(x)dx$.

When using this formula, it is required that the plane shape contains this axis and rotates around it, so that the obtained volume can be directly applied to the formula. However, if the plane shape does not contain a coordinate axis and rotates around this axis, the formula cannot be directly applied. In order to use the derived formula, it is necessary to construct conditions that meet the formula, for example, finding the volume of a rotating body that a circle $(x-b)^2 + y^2 = a^2 (0 < a < b)$ rotates around the y axis. From the image, it can be seen that the planar figure does not include the y axis. In order to use formulas, teachers guide students to carefully consider how to include y axis in the graphics during teaching. After analysis, it can be found that the plane figure enclosed by the right semicircle, straight $y = a$, straight $y = b$, and y axis rotates around y axis. The volume of such a rotating body can be applied to the formula derived. Similarly, the other part is the left semicircle, which is a curved trapezoid rotating around the y axis. The difference in volume between these two parts is the volume of the rotating body to be calculated.

6. USING INFINITESIMAL ANALYSIS TO CALCULATE THE VOLUME OF A CURVED TOP COLUMN

If what is given is not a spatial curve, but a spatial surface, how to find the volume of a solid enclosed by a spatial surface and a plane? Assuming that the plane of the desired volume is a bounded region D, the function $f(x,y)$ is a first-order partial derivative with continuity on the bounded region D. Infinitesimal analysis is still be considered. The projection area of the entire curved top cylinder on xoy surface is D. Selecting a small closed area $d\sigma$ containing point $P(x, y)$ within D as the base area of the small area, the volume of a small curved top cylinder can be approximated as high by $z = f(x, y)$. The volume of a small flat topped cylinder with $d\sigma$ as its base, that is, $dV = f(x, y)d\sigma$. Integrating on a regional D yields the desired volume of the solid.

7. CONCLUSION

With the in-depth study of multivariate calculus, the importance of infinitesimal analysis in modern integral engineering is becoming increasingly high. Through practical application analysis, students can have a deeper understanding of the application of infinitesimal analysis. Through the flexible application of infinitesimal analysis in univariate functions, it is believed that students can apply infinitesimal analysis flexibly to problems such as multiple integrals, liquid pressure, gravity, and moment of inertia.

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